Algebra Insight: Algebraic Transformations 1 and 2

Focus on fluent recognition of equivalent expressions and equations across algebraic transformations.

Algebraic Transformations 1 and 2 is a sequenced pair of PLMs that are intended to build fluency with the following:

- Analysis of structure in algebraic equations
- Maintaining equivalence over transformations, including adding or subtracting constants, expanding expressions, and collecting like terms.
- Working with rational numbers.

Algebraic Transformations 1 and 2 teach students to see patterns and relationships in algebraic equations; more specifically, it enables learners to be able to quickly recognize equivalent expressions and legal algebraic transformations. The basic rules of algebra do not seem too difficult: whatever you do to one side of the equation you must do to the other. The problem is not so much a lack of understanding of the procedure, but instead an inability to see what these operations are actually doing, how they are related to each other, and how they alter the structure of the underlying equation. Simply knowing the rule is not enough; students need to develop fluency and familiarity in recognizing the properties of equality and logical equivalence as equations and expressions undergo transformations. Algebraic Transformations aims to remedy this often overlooked aspect of algebra learning. A majority of each learning session is devoted to structure mapping problems. Students are presented with an equation and are asked to identify from a set of four choices an equivalent form of the equation. The task may seem difficult and strange to students at first, since they are not asked to actually solve anything. However, over time, these activities help engage students' natural pattern recognition skills in noticing key structural features of equations and legal transformations, while at the same time filtering out characteristics of illegal operations. The learning sessions are interspersed with more conventional solve problems in which students must decide what operation to perform in order to change an equation from one state to another, including solving for a variable. Together, these problem types provide a unique and powerful learning experience that results in long-lasting benefits in mastering algebra.

Algebraic Transformations 1 involves simpler problems. Transformations are limited to one step and do not include rational expressions**. Algebraic Transformations 2** involves more complicated expressions including ones with rational expressions and a greater number of variables and transformation steps. And, of course, like all of our PLMs, both versions adapt to each individual student, using his or her own performance data to pace and sequence the learning process and guide the learner to fully certified mastery, based on both accuracy and fluency criteria.

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Alignment to Common Core Mathematics Standards

Standards for Mathematical Practices

- *Make sense of problems and persevere in solving them.*
- *Reason abstractly and quantitatively.*
- *Look for and make use of structure.*
- *Look for and express regularity in repeated reasoning.*

Common Core Mathematics Standards by Domain & Grade	
EXPRESSIONS AND EQUATIONS	
Grade 6.EE: Expressions and Equations	• Apply and extend previous understandings of arithmetic to algebraic expressions. ○ Identify when two expressions are equivalent, regardless of which value is substituted into them.
Grade 7.EE: Expressions and Equations	• Use properties of operations to generate equivalent expressions. • Solve problems using numerical and algebraic expressions and equations. ○ Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate.

Grade 8.EE: Expressions and Equations	• Analyze and solve linear equations.
	  ○ Successively transform the equations into simpler forms.
	  ○ Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

THE NUMBER SYSTEM	

Grade 7.NS: The Number System	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
	  ○ Understand subtraction of rational numbers as adding the additive inverse.
	  ○ Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers.

High School A-SSE: Seeing Structure in Expressions	**ALGEBRA**
	• Interpret the structure of expressions.
	  ○ Use the structure of an expression to identify ways to rewrite it.
	• Write expressions in equivalent forms to solve problems.
	  ○ Choose and produce an equivalent form of an expression.

High School A-CED: Creating Equations	• Create equations that describe numbers or relationships. ○ Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
Note: Where the PLM provides partial coverage of a standard, the relevant part of the standard is cited.	