Core Math Insight: Operations and Algebraic Thinking

The Best Way Ever for Students to Learn Crucial Basic Number Facts!

- > Teach basic Addition, Subtraction, Multiplication and Division
- > Teach one operation at a time or select multiple operations
- Make the connection to word problems and meaningful use of number facts and operations
- Build a foundation for algebraic thinking from a sound base of fluent arithmetic and number knowledge
- Build number system knowledge and fluency with factors

This Core Math Insight PLM leads each student to fluent mastery with:

- Number fact families
 - o Addition and Subtraction (within 20)
 - Multiplication and Division (with factors from 0 through 12)
- Word problems: representing and solving a full range of word problems involving three whole numbers for addition, subtraction, multiplication and division
- Factor pairs for numbers through 100

The CMI: Operations and Algebraic Thinking PLM builds on principles of learning and memory to help each student develop long-lasting fluency with basic number facts and operations in the most efficient and effective way possible. Students will be able to quickly and accurately recall math facts, but they will also gain far more. Following the recommendation of the National Research Council's Mathematics Learning Study Committee, highlight which emphasizes the relational character of number combinations, this PLM deliberately presents intuitive opportunities for students to recognize important patterns and relations among basic math facts in order to provide a strong foundation for algebraic thinking. The module includes word problems involving whole number quantities that engage students in quantitative reasoning, in which they must solve word problems by appropriately relating quantities and operations. The learning set draws on a comprehensive set of fundamental word problem types, including joining, separating, part-part-whole, and comparing problems for addition and subtraction, as well as multiplication, measurement (quotitive) division, and sharing (partitive) division problems. Feedback on problems helps students refine their ability to model word problems mathematically and to reach mastery.

The software can be customized for students at different levels by selecting the operation types that are included and the difficulty level. Other settings enable a focus on word problems and factor problems. For example, younger children can limit the learning set to addition and subtraction problems between 0 and 10 with few word problems, while older students could work with all operations and problem types including a full set of word problems. And, of course, like all of our PLMs, the software adapts to each individual student, using his or her own performance data to pace and sequence the learning process and guide the learner to fully certified mastery, based on both accuracy and fluency criteria.

¹ National Research Council. (2001). *Adding It Up: Helping Children Learn Mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

² Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., and Empson, S. B. (1999). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

Alignment to Common Core Mathematics Standards

Standards for Mathematical Practices

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Common Core Mathematics Standards by Grade	
Domain: Operations and Algebraic Thinking	
Grade 1*	 Represent and solve problems involving addition and subtraction. Understand and apply properties of operations and the relationship between addition and subtraction. Add and subtract within 20. Demonstrate fluency for addition and subtraction within 10. Work with addition and subtraction equations. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.
Grade 2	 Represent and solve problems involving addition and subtraction Add and subtract within 20. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
Grade 3	 Represent and solve problems involving multiplication and division. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. Understand properties of multiplication and the relationship between multiplication and division. Fluently multiply and divide within 100. Solve problems involving the four operations.
Grade 4	 Use the four operations with whole numbers to solve problems. Gain familiarity with factors and multiples. Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors.

^{*} Note that these Grade 1 standards are implemented in the context of a computer program that requires the ability to read math problems independently on a computer screen. Most first graders will start with a more scaffolded context, such as having an adult verbalize the problems or modeling problems using manipulatives. As children become able to work more independently, the PLM is a terrific tool to help them transition to solving problems using mental strategies and number knowledge. For older students, the PLM facilitates the development of algebraic thinking from fluent arithmetic.